Functions

Fact — A function is a *mapping* from one set to another set.

Example

Find the range of the function $f(x) = x^2 - 2x + 4$ with domain $-1 \leq x \leq 2$.

Example

A function is defined as $f(x) = \frac{2x}{x^2+1}$.

(a) By letting $y = f(x)$, show that $yx^2 - 2x + y = 0$.

(b) Using the discriminant of a quadratic, find the range of values for y such that (a) has real roots.

(c) Deduce the range of the function $f(x)$ and hence sketch the curve $f(x)$

Composition of Functions

Suppose we have

$$f : A \to B$$
$$g : B \to C$$

then it makes sense to talk about $gf = g \circ f$, which is defined by:

If $A = B = C$, it can make sense to talk about both fg and gf, for example $\sin : \mathbb{R} \to \mathbb{R}, x \mapsto x^2 : \mathbb{R} \to \mathbb{R}$, so $\sin(x^2)$ and $\sin^2(x)$ are both valid compositions of these functions.

Example

The functions f and g are given by:

$$f(x) = 5e^{-x} + 1, x \in \mathbb{R}, x \geq 0$$
$$g(x) = 2x + 1, x \in \mathbb{R}$$

(a) Find...

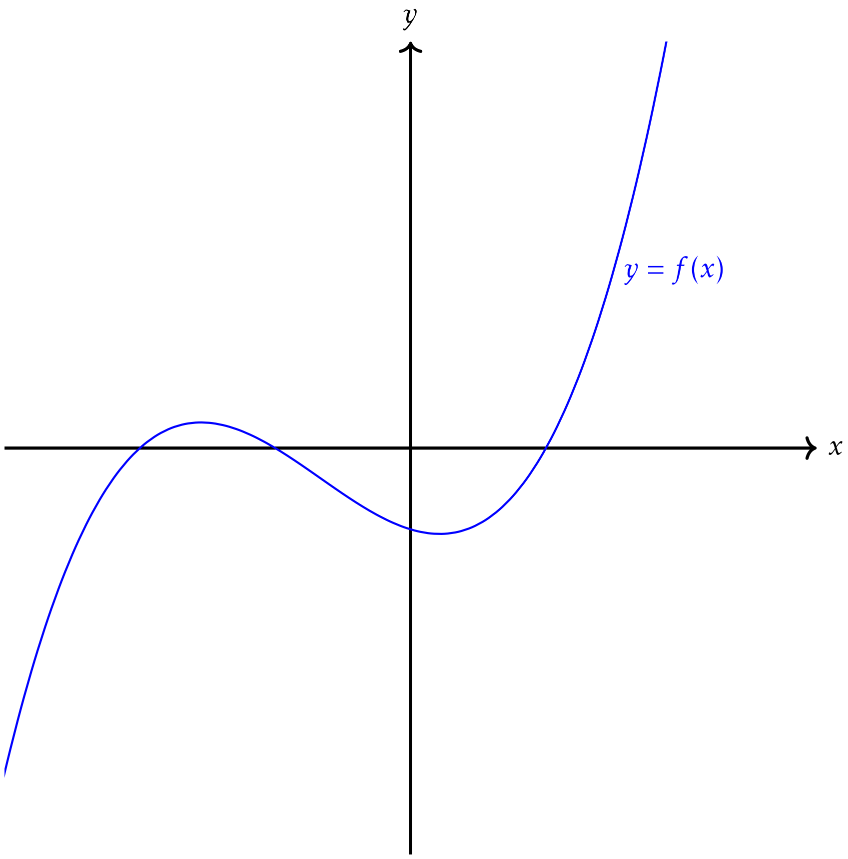
(i) ... an expression for $gf(x)$

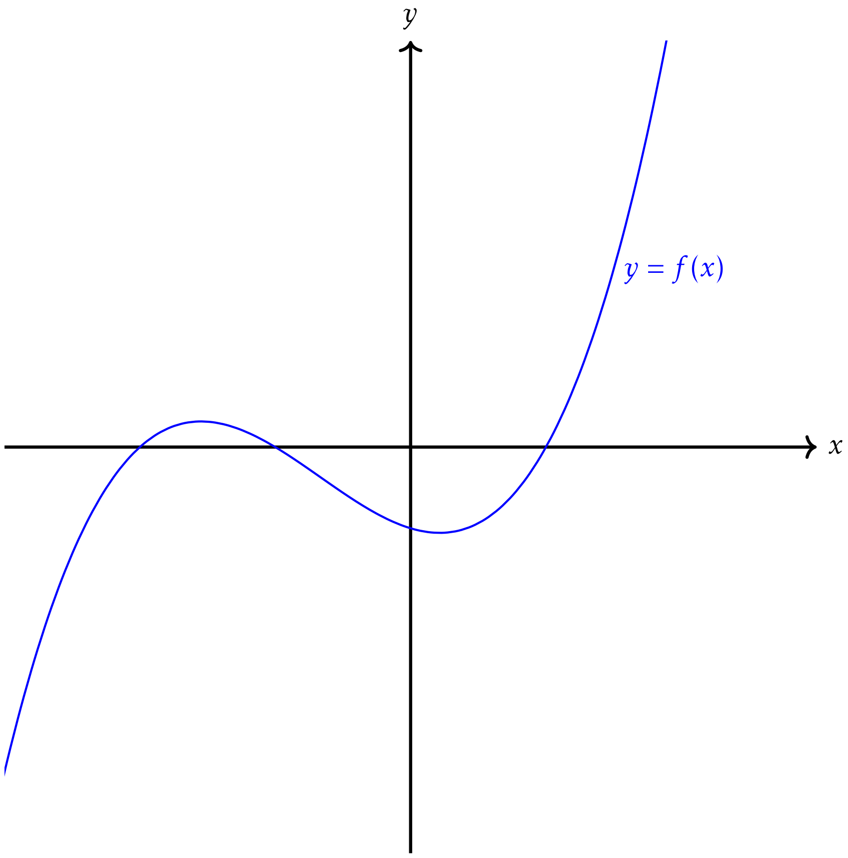
(ii) ... the range of $gf(x)$

(iii) ... the domain of $fg(x)$

(b) Show the only solution of the equation $fg(x) = 5e^{2x+1} - 9$ can be written as $x = \frac{1}{2}\left[-1 + \ln(1 + \sqrt{2})\right]$

Transformation of functions





$f(x) \mapsto$	Transformation
$f(x+a)$	translation by $\binom{-a}{0}$
$f(x)+a$	translation by $\binom{0}{a}$
$f(ax)$	stretch by scale factor $\frac{1}{a}$ parallel to x-axis
$af(x)$	stretch by scale factor a parallel to y-axis
$f(-x)$	reflection in y-axis
$-f(x)$	reflection in x-axis